Example – Conversion from CFG to CNF.

Eg:

$S \to ASA \mid aB$
$A \to B \mid S$
$B \to b \mid \varepsilon$

1. S is present in RHS. Introduce a new start symbol.

$S' \to S$
$S \to ASA \mid aB$
$A \to B \mid S$
$B \to b \mid \varepsilon$

2. Null production removal

$\left.\begin{array}{l} A \to B \to \varepsilon \\ B \to \varepsilon \end{array}\right\}$ Nullable variable $= \{A, B\}$

$S' \to S$
$S \to ASA \mid aB \mid SA \mid AS \mid a$
$A \to B \mid S$
$B \to b$

Unit production

$S' \to S \to$ ASA, aB, SA, AS, a

$A \to B \to b$

$A \to S \to$ ASA, aB, SA, AS, a

$S' \to ASA \mid aB \mid SA \mid AS \mid a$
$S \to ASA \mid aB \mid SA \mid AS \mid a$
$A \to b \mid ASA \mid aB \mid SA \mid AS \mid a$

$B \to b$

All useful symbols as all derive a terminal and are reachable from start state.

3. $V \to V \cdot V$
$V \to T$
$S' \to \varepsilon$

$S' \to ASA \mid aB \mid SA \mid AS \mid a$
$S \to ASA \mid aB \mid SA \mid AS \mid a$
$A \to b \mid ASA \mid aB \mid SA \mid AS \mid a$
$B \to b$

↓

$S' \to AC \mid aB \mid SA \mid AS \mid a$
$S \to AC \mid aB \mid SA \mid AS \mid a$
$A \to b \mid AC \mid aB \mid SA \mid AS \mid a$
$B \to b$
$C \to SA$

↓

$$\left.\begin{array}{l} S' \to AC \mid DB \mid SA \mid AS \mid a \\ S \to AC \mid DB \mid SA \mid AS \mid a \\ A \to b \mid AC \mid DB \mid SA \mid AS \mid a \\ B \to b \\ C \to SA \\ D \to a \end{array}\right\} \text{CNF}$$

eg: $S \to ASB \mid b$
$A \to aAS \mid a \mid \varepsilon$
$B \to SbS \mid A \mid bb$

1. $S' \to S$
$S \to ASB \mid b$
$A \to aAS \mid a \mid \varepsilon$
$B \to SbS \mid A \mid bb$

2. nullable variable: $\left.\begin{array}{l} A \to \varepsilon \\ B \to A \to \varepsilon \end{array}\right\} \{A, B\}$

$S' \to S$
$S \to ASB \mid SB \mid AS \mid b$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid A \mid bb$

unit productions

$S' \to ASB \mid SB \mid AS \mid b$
$S \to ASB \mid SB \mid AS \mid b$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid aAS \mid a \mid aS \mid bb$

$B \to A \to aAS$
$\quad \to a$
$\quad \to aS$

useless X

3. $S' \to ASB \mid SB \mid AS \mid b$
$S \to ASB \mid SB \mid AS \mid b$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid aAS \mid a \mid aS \mid bb$

$S' \to CB \mid SB \mid AS \mid b$
$S \to CB \mid SB \mid AS \mid b$
$A \to aC \mid a \mid aS$
$B \to SbS \mid aC \mid a \mid aS \mid bb$
$C \to AS$

$S' \to CB \mid SB \mid AS \mid b$
$S \to CB \mid SB \mid AS \mid b$
$A \to DC \mid a \mid DS$
$B \to SbS \mid DC \mid a \mid DS \mid bb$
$C \to AS$
$D \to a$

$S' \to CB \mid SB \mid AS \mid b$
$S \to CB \mid SB \mid AS \mid b$
$A \to DC \mid a \mid DS$
$B \to SG \mid DC \mid a \mid DS \mid FF$
$C \to AS$
$D \to a$
$F \to b$
$G \to FS$

CNF

Greibach Normal form (GNF):

$V \to T$
$V \to TVVV\ldots$
$S \to \varepsilon$ (epsilon)

$A \to a$
$A \to aBCD\ldots$

Eg:

$S \to aA \mid bB$
$B \to bB \mid b$
$A \to aA \mid a$

✓ GNF

$S \to aA \mid bB$
$B \to bB \mid \varepsilon$
$A \to aA \mid \varepsilon$

GNF ✗

- for a given grammar, more than 1 GNF is possible.
- language generated by GNF & by CNF should be same.

Conversion from CFG to GNF:

1. Convert grammar to CNF.
2. If left recursion exists, remove it
3. Convert productions to GNF.

$NT \to T$

$NT \to T.NT.NT....$

$S \to \varepsilon$

Left Recursion:

- Production in which left most symbol of RHS = symbol present on LHS.

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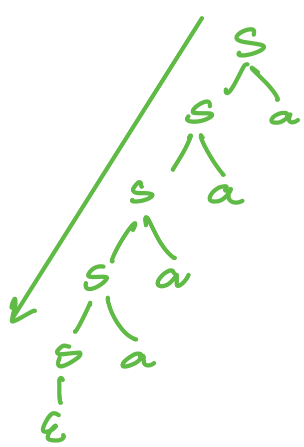
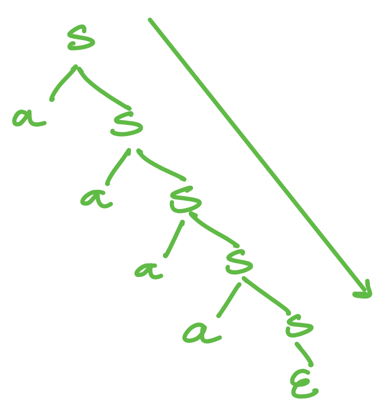
- Grammar having a production with left recursion, such a grammar is called as left recursive grammar.

$S \to Sa \mid \varepsilon$



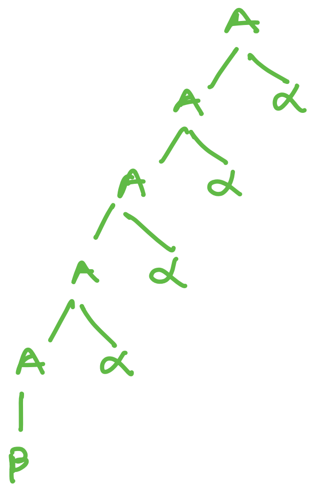
$S \to aS \mid \varepsilon$



Remove left recursion:

$A \to A\alpha \mid \beta$

Conversion Left Recursion to Right Recursion



Language: $\beta\alpha^*$ new production

$A \to \beta A'$

$A' \to \alpha A' \mid \varepsilon$

$A \to A\alpha \mid \beta_1 \mid \beta_2 \mid \beta_3 \cdots \beta_k$

$\boxed{\begin{array}{l} \beta_1\alpha^* \\ \beta_2\alpha^* \\ \beta_3\alpha^* \\ \vdots \\ \beta_k\alpha^* \end{array}}\ A'$

$A \to \beta_1 A' \mid \beta_2 A' \mid \beta_3 A' \cdots \mid \beta_k A'$

$A' \to \alpha A' \mid \varepsilon$ } α^* generate

Eg: Convert CFG to GNF?

$S \to XB \mid AA$

$A \to a \mid SA$

$B \to b$

$X \to a$

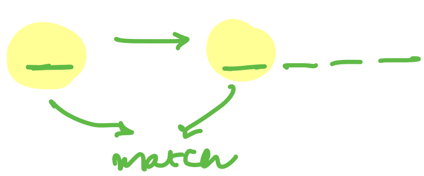
1. GNF?

$V \to T$

$V \to VV$

$S \to \varepsilon$ } already true

2. Left Recursion?



3.

$S \rightarrow xB \mid AA$
$A \rightarrow a \mid SA$
$B \rightarrow b$
$x \rightarrow a$

$\downarrow$

$S \rightarrow xB \mid AA$
$A \rightarrow a \mid xBA \mid AAA$
$B \rightarrow b$
$x \rightarrow a$

$\downarrow$

$S \rightarrow aB \mid AA$
$A \rightarrow a \mid aBA \mid AAA$
$B \rightarrow b$
$x \rightarrow a$

$\downarrow$ Remove left recursion

$A \rightarrow A\alpha \mid \beta_1 \mid \beta_2$

$\downarrow$

$A \rightarrow \beta_1 A' \mid \beta_2 A'$
$A' \rightarrow \alpha A' \mid \varepsilon$

$A \rightarrow \underbrace{a}_{\beta_1} \mid \underbrace{aBA}_{\beta_2} \mid A\underbrace{AA}_{\alpha}$

$A \rightarrow aA' \mid aBAA'$
$A' \rightarrow AAA' \mid \varepsilon$

$S \rightarrow aB \mid AA$
$A \rightarrow aA' \mid aBAA'$
$A' \rightarrow AAA' \mid \varepsilon$
$B \rightarrow b$
$x \rightarrow a$

$\downarrow$ Remove ε production

$S \rightarrow aB \mid AA$
$A \rightarrow aA' \mid aBAA' \mid a \mid aBA$
$A' \rightarrow AAA' \mid AA$
$B \rightarrow b$
$x \rightarrow a$

$\downarrow$

GNF
$S \rightarrow aB \mid aA'A \mid aBAA'A \mid aA \mid aBAA$
$A \rightarrow aA' \mid aBAA' \mid a \mid aBA$
$A' \rightarrow aA'AA' \mid aBAA'AA' \mid aAA' \mid aBAAA' \mid aA'A \mid aBAA'A \mid aA \mid aBAA$
$B \rightarrow b$